Inventory-Based Sensitivity Analysis of the Large Tree Diameter Growth Submodel of the Southern Variant of the Forest Vegetation Simulator

Giorgio Vacchiano[1]
John D. Shaw[2]
R. Justin DeRose[3]
James N. Long[3]

Abstract—Diameter increment is an important variable in modeling tree growth. Most facets of predicted tree development are dependent in part on diameter or diameter increment, the most commonly measured stand variable. The behavior of the Forest Vegetation Simulator (FVS) largely relies on the performance of the diameter increment model and the subsequent use of predicted dbh in forecasting tree attributes.

Previous research has shown the efficacy of localized inventory data in calibrating model parameters when better predictions of individual and stand growth in focal geographic areas are sought. A sample-based sensitivity analysis (SA) is proposed as a preliminary step to model calibration, in order to identify which variables are most influential in determining predicted outcomes. SIMLab software was used for SA of the default dbh increment submodel in FVS-SN; samples were obtained from a recent inventory of longleaf pine stands in Fort Bragg, NC. Preliminary results show that dbh is by far the most important variable, followed by site index and competition-related predictors. Topographical and other site variables were largely non-influential. Before calibration and re-engineering of the submodel, variables conveying redundant or non-influential information may be considered for elimination.

Introduction

Project Background

The Fort Bragg military installation is located 10 miles northwest of Fayetteville, North Carolina, in the Sandhills Region. Of the 161,597 total acres, an estimated 65,000 are covered by longleaf pine (*Pinus palustris* Mill.) dominated forests. Habitat recovery efforts for the endangered red-cockaded woodpecker (*Picoides borealis*) currently are a priority at Fort Bragg (Blythe and others 2001). Forest inventory and monitoring are needed to assess suitability of forest conditions to the species' habitat requirements (U.S. Fish and Wildlife Service 2003), as well as to provide indicators of overall ecosystem integrity and capability of lands to support military training operations.

A 10-year forest inventory program is currently implemented throughout the installation; in addition, forest stands are annually monitored to update changes resulting from natural growth and silviculture treatments. In order to plan for future growth of the forest and development of military facilities, 10-year growth projections at the stand level were formulated for the entire installation at the time of the first inventory. However, model-based simulations provided unrealistically high stocking levels, and preliminary testing of the Southern Variant (Donnelly and others 2001) of FVS (FVS-SN) showed a similar tendency.

The main reason for such discrepancy has been speculated as being related to an erroneous representation of the inherent maximum size-density boundary for key forest species (Shaw and Long 2007). This issue cannot be adequately solved by standard model re-fitting techniques; DeRose and others (this proceedings) proposed a modification to FVS program logic that would yield more accurate survival predictions, in accordance with the findings by Shaw and Long (2007). However, Fort Bragg spans over an area much smaller than the one referenced by developers of FVS-SN (see after). For this reason, we put into question the validity of all components of the SN model, under the hypothesis that discrepancies between local growing conditions and the more general relationships outlined by the variant might prompt growth prediction errors at the

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[1] Graduate Student, Dipartimento Agroselviter, Università di Torino, Grugliasco, Italy; e-mail: giorgio.vacchiano@unito.it.

[2] Analyst, USDA Forest Service, Rocky Mountain Research Station, Forest Inventory and Analysis, Ogden, UT; e-mail: jdshaw@fs.fed.us.

[3] Graduate student, and Professor, respectively, Utah State University, Department of Wildland Resources, Logan, UT; e-mail rjderose@cc.usu.edu; fakpb@cc.usu.edu.

individual tree scale. A research effort is currently underway which aims to evaluate and refit FVS-SN using forest inventory data collected on Fort Bragg (Shaw and others 2006). This paper represents a first step using the base FVS-SN submodels in order to establish how Fort Bragg data look in relation to the submodels specified in SN over a much wider geographic range, and thus calibrated over a much different dataset.

The Southern Variant: Features and Challenges

FVS-SN was developed from Forest Inventory and Analysis (FIA) data, Forest Service research data, and data from the Bureau of Indian Affairs. Its geographic coverage spans most of the southeastern United States (Donnelly and others 2001). Growth relationships for such a wide area are refined with the help of species-specific coefficients for each submodel equation. All submodels portray average growing conditions and allometric relationship throughout the southern states. Additionally, diameter increment and standing volume computations also include location codes accounting for the region, National Forest, and Ranger District where the stand is situated, and Ecological Unit Codes (Keys and others 1995) at the province level as a mean of distinguishing between major geographic areas within the region.

Even if the model includes a self-calibration feature, allowing it to adjust diameter and height growth predictions based on field increment data (Dixon 2002) there are grounds to suspect that local variability is not adequately reflected. Developers of FVS-SN stated that “If further research and/or evidence shows that tree growth differences are distinguishable at finer scales, such results can be fit into the growth relationships” at subsequent time (Donnelly and others 2001). Therefore, ecological subdivisions at a scale smaller than Province level may in some cases be proven to have an effect on diameter change computations.

Since the first version of Prognosis (Stage 1973), diameter growth prediction has represented the key modeling function, upon which other submodels depend, at least in part, for their inputs. In FVS-SN the diameter growth submodel for large trees, i.e., those with a diameter at breast height (dbh) greater than 3 inches, uses a 14-coefficient equation with a mixture of categorical and continuous variables (table 1). The dependent variable is the logarithm of the predicted periodic change in squared inside-bark diameter (Wykoff and others 1982).

When this equation was fitted to the Fort Bragg data in its complete form, three potential problems emerged. First, the regression yielded relatively low R^2 values. Second, some coefficients were found to have unrealistic signs, for example, competition-related variables with a positive effect on growth. Both anomalies have been previously related to correlation problems and the degree of variability in a given data set (Neter and others 1990); nevertheless, FVS-SN developers stated that “detection of multicollinearity was a major effort in picking independent variables for the diameter increment submodel of FVS-SN” (D. Donnelly, personal communication), which rules out interconnected distributions of independent variables as a source of error. Third, since the ranges of some variables are relatively small on Fort Bragg as compared to the variability found within the geographic range encompassed by FVS-SN, we anticipated that some input factors might be redundant or even unnecessary components of the submodel at the local scale.

Sensitivity Analysis of Model Output

In order to assess and rank the role of each independent variable in predicting diameter increment of longleaf pine on Fort Bragg, we carried out a sensitivity analysis (SA) of model output on the diameter increment submodel of FVS-SN. Innis (1979) defined SA as “the systematic search for those model entities to which the model is most sensitive”; the terms “model entities” refers to the measurement accuracy of input factors, the value of the parameters used by the model (Herring 2007), as well as the model form itself. The effect of incremental inclusion of independent variables and the effect of changes in functional relationships may be assessed both at the submodel and at the model superstructure level. However, the most general use of SA is concerned with model simplification (Saltelli and others 2008). The objective is to identify the factor or the subset of input factors that can be fixed at any given value over their range of uncertainty without reducing significantly the output variance. Regardless of their contribution to model predictions, insensitive model components need neither to be measured with great

Table 1—Variables and description in the FVS diameter growth submodel (from Donnelly and others 2001). Input variables account for the growth potential of individual trees, the influence of the tree's neighbors and the site's ability to support growth.

	Variable	Description
ln(dds)[a] =	b_0	intercept
	$+ b_1 \cdot \ln$ dbh	log of dbh (at beginning of estimation period)
	$+ b_2 \cdot$ dbh^2	squared dbh
	$+ b_3 \cdot \ln$ crwn	log of percent crown ratio
	$+ b_4 \cdot$ hrel	relative height
	$+ b_5 \cdot$ SI	site index for the species
	$+ b_6 \cdot$ plttba	plot basal area
	$+ b_7 \cdot$ pntbal	plot basal area in trees larger than subject tree
	$+ b_8 \cdot$ tan slp	tangent of slope in degrees
	$+ b_9 \cdot$ f cos	tangent of slope, cosine of aspect
	$+ b_{10} \cdot$ f sin	tangent of slope, sine of aspect
	$+ b_{11} \cdot$ fortype	categorical variable for forest type group
	$+ b_{12} \cdot$ ecounit	categorical variable for ecological unit group
	$+ b_{13} \cdot$ plant	categorical variable for planted stands

[a] dds = (diameter inside bark at time$_0$ + periodic diameter growth)^2 – diameter inside bark^2 (Wykoff and others 1982).

precision nor to be scrutinized during refitting of the model. Since their behavior is closer to that of constants than of variables, they might be omitted for the sake of parsimony should the model be reworked under a different form. Conversely, it is useful to know about model components with high sensitivity, because these have the greatest impact on model predictions (Vanclay and Skovsgaard 1997) and might need to be measured or assessed with greater care.

Most SA approaches to date have relied on local SA, i.e., the evaluation of the effect exerted on model outputs by individually varying only one of the model inputs across its entire range of plausible values, while holding all other inputs constant (Cullen and Frey 1999). A major drawback of this method is that interactions between input variables cannot be computationally taken into account. Thus, the results of nominal range sensitivity analysis are potentially misleading, especially for multilinear and nonlinear models (Frey and Patil 2002).

Hamilton (1997) proposed what he called "sensitivity analysis" of the FVS suite as a whole. His method was based on *a priori* alteration of submodel output, by means of FVS keywords such as BAIMULT, HTGMULT and MORTMULT (Van Dyck 2001). The percent difference in selected stand descriptors at the end of the modeling time step, resulting from the introduction of fixed perturbations in each of the submodels, represented the author's chosen sensitivity metric. However, this approach was affected by limitations similar to one-factor-at-a-time analysis.

We propose the use of first-order sensitivity indices, which assess the variance of model output Y due to model input X_i (Saltelli and others 2004). Our specific aim is to assess which of the input factors are most influential on the large-tree diameter growth submodel.

Methods

Although several techniques have been proposed (Frey and Patil 2002), sampling-based approaches to uncertainty and sensitivity analysis are both effective and widely used. Analyses of this type involve generating, via Monte Carlo simulations, a set of model evaluations Y_i (i = 1… N), corresponding to N different sampled values X_i of the vector $X = f(X_1, X_{2,\dots} X_k)$ of k input factors, and subsequently mapping uncertain analysis inputs to uncertain analysis results. The steps involved in conducting such an effort are the following (Helton 2005):

- Definition of probability distributions to characterize uncertainty in analysis inputs;
- Generation of samples from uncertain analysis inputs;
- Propagation of sampled inputs through model simulation;

- Assessment of uncertainty analysis results; and
- Determination of sensitivity analysis results.

Since we were interested in model parsimony, rather than in assessing error propagation through the model, we chose to consider only stochastic uncertainty, i.e., that arising from the behavioral properties of the system under study. Therefore, we adopted the default FVS-SN dbh increment submodel as the function to evaluate, retaining its original parameterization and evaluating uncertainty of each input factor across its potential variability in the inventory.

Growth data from 7,302 individual longleaf pines were available from Fort Bragg forest inventory and were used to infer the shape, statistical properties (estimates of population mean and standard deviation) and range of each factor's probability density function (PDF) (table 2). PDFs of sample variables were positively tested for normality by means of one-variable Kolmogorov-Smirnov test ($p < 0.05$) and truncated to minima and maxima measured in the field to avoid sampling outliers. Variables such as slope and forest type coding were assigned a discrete PDF with classes and weights inferred from sample frequencies. Biologically relevant correlations between input factors (tree dbh and height, tree height and crown ratio, crown ratio and stand basal area, and between stand basal area and plot basal area) were computed by means of Pearson's coefficients and their value entered in a dependence tree structure (Meeuwissen and Cooke 1994) (table 3).

Next, we generated an iterated sample of elements from the distribution of the inputs previously specified. Latin hypercube, or n-dimension stratified sampling, was chosen because of its efficient stratification properties allowing for the extraction of a large amount of uncertainty and sensitivity information with a relatively small sample size (Helton and Davis 2003). Moreover, this technique performs better than simple random sampling when the output is dominated by a few input factors (Iman and others 1981).

SIMLab software (EU IPSC 2004) was used for all steps of SA; the software architecture is represented in figure 1. The randomized sample is generated in SIMLab using an iterative function based on a user-defined seed number. We instructed the software to generate 10,000 samples, a number close to the number of tree records used for the default parameterization of FVS-SN in longleaf pine (Donnelly and others 2001) but much higher than the suggested minimum (McKay and others 1979). The generated sample served as a starting point for Monte Carlo-based model runs; in the model execution

Table 2—Characterization of the input factors for sensitivity analysis of the diameter increment submodel.

Input	Definition	PDF shape	Range	Units	Notes
D	Diameter at breast height	Normal	2–30	inches	
CR	Live crown ratio	Normal	1–00	percent	
H	Tree height	Normal	10–101	feet	For relative height computation
H40	Height of 40 thickest trees ac^{-1}	Normal	40–103	feet	
SI	Site Index	Normal	44–132	feet	
BA	Basal area (stand)	Normal	5.5–158	$feet^2\ ac^{-1}$	
pointBA	Basal area (plot)	Normal	10–270	$feet^2\ ac^{-1}$	For point BA in larger trees computation
rank	percentile of tree's dbh in plot	Uniform	0–1	-	
slope	plot mean slope	Discrete	0–0.8	rad	
aspect	plot mean aspect	Uniform	0–2π	rad	
EUC	Ecological unit code	Constant	0	categ.	PVP232
forcode	Forest cover type	Discrete	0–1	categ.	From Donnelly and others (2001)l
plant	Plantation origin	Constant	0	binary	None in Fort Bragg

Table 3—Correlation between input factors as measured from Fort Bragg inventory data.

Variable 1	Variable 2	Pearson's R
dbh (inches)	Height (feet)	0.69
Height (feet)	Live crown ratio	–0.34
Live crown ratio	Stand basal area ($feet^2\ ac^{–1}$)	0.35
Stand basal area ($feet^2\ ac^{–1}$)	Plot basal area ($feet^2\ ac^{–1}$)	0.56

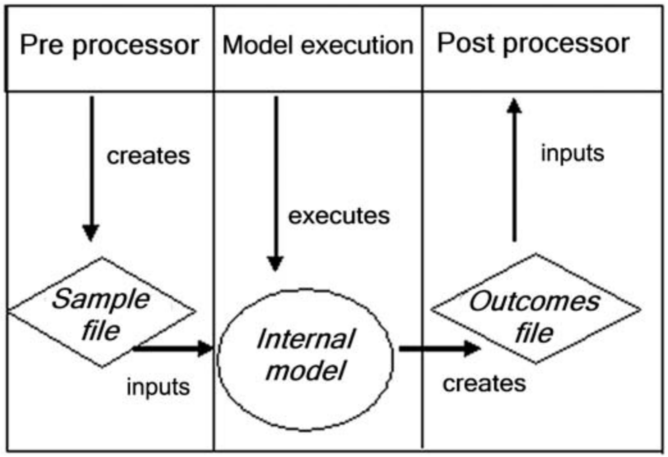

Figure 1—Internal model execution process in SIMLab (modified from EU IPSC 2004).

phase, each element of the sample is supplied to the model as input, and the corresponding model predictions are saved for lat uncertainty and sensitivity analyses, performed by the statistical post processor.

Uncertainty analysis aimed at comparing the PDF of generated diameter increment values with the ones measured in the field. Field measurements, inventory protocols and data treatment are described by Shaw and others (2006).

The outputs whose sensitivity was evaluated were both *dds*, the change in squared inside-bark diameter (in^2) during the estimation period, and d_g, the value of inside-bark diameter increment after a 5-year simulation cycle, as computed by the following:

$$d_g\,(inches) = \sqrt{dib^2 + dds} - dib \qquad [1]$$

where *dib* is tree dbh inside bark at the beginning of the modeling period (inches). A constant ratio of 1.15 has been adopted as the bark thickness coefficient for longleaf pine on Fort Bragg, independent of tree size or age (R.J. DeRose, unpublished data).

Sensitivity indicators were represented by standardized regression coefficients (SRC), that quantify the change in Y associated with a unit of change in a given parameter X_i, all other parameters remaining constant (Draper and Smith 1988; Helton 1993). The rank-based version of the index was used in order to account for nonlinearity in the model (Saltelli and others 2000). Finally, sensitivity tests based on data partitioning such as the Smirnov two-sample test (Conover 1980) helped assess the importance of each input factor. The test splits the sample space for factor X_i into two subsamples according to the quantiles of the output distribution Y. If the distributions of the two subsamples can be proven different (index values closer to 1) then the factor X_i is considerer influential. The influence of input factors on model output was computed separately for four different dbh size classes. Independent variables were entered in the model in base rather than composite form (for example, relative height has been split to tree height and height of the 40 largest trees per acre).

Results and Discussion

Mean modeled d_g was 0.54 ± 0.11 inches (modeling step: 5 years), a value statistically different (two-sample t-test, $p < 0.0001$) but close to the average 5-year dbh increment measured on longleaf pine increment cores in the 2000 inventory (0.60 ± 0.30 inches). Nevertheless, modeled output is characterized by a much lower uncertainty than measured data (fig. 2), the latter having a wider and more skewed distribution (range: 0.08 to 2.58 inches, skewness = +1.565). We hypothesized the lower variability of modeled growth was due to a higher homogeneity of tree measurements used for original FVS-SN calibration. However, this was inconsistent with the fact that the default model presents a much better goodness-of-fit to SIMLab-generated Fort Bragg data than to the original calibration dataset (R^2: 0.94 and 0.52 respectively).

A certain degree of model-induced simplification was not unexpected. The slight overprediction at the lower end of the dbh increment range is not likely to be problematic,

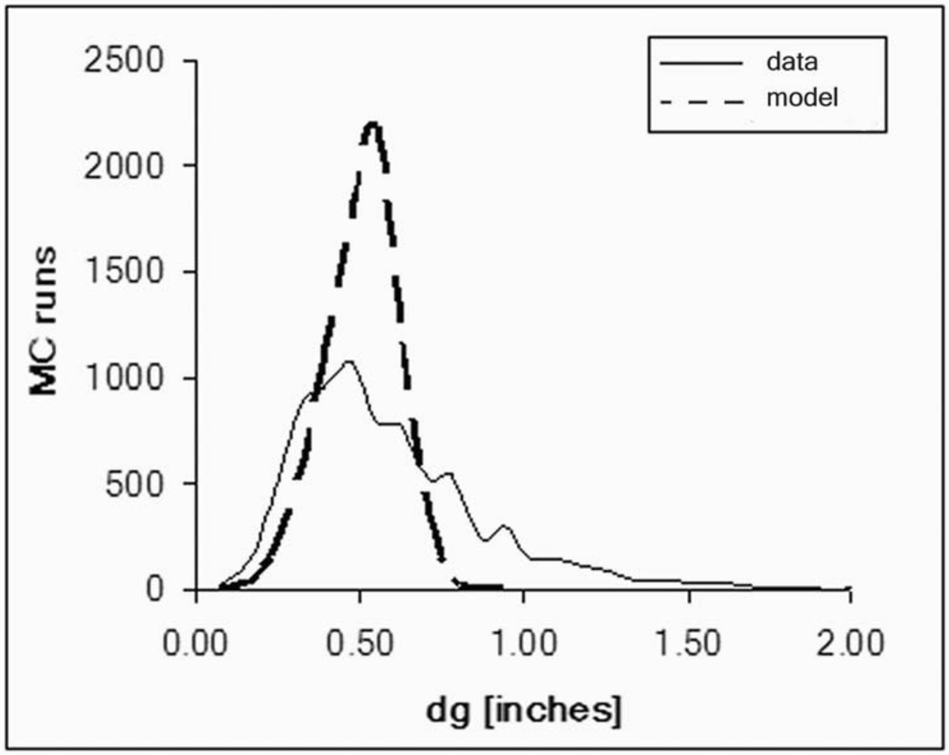

Figure 2—Probability density function of variable d_g (5-year diameter growth) resulting from uncertainty analysis (10,000 Monte Carlo simulations) as compared to that measured in the field.

and may be explained by the presence of a few old trees (ages ≥ 100 years), that likely represent leftovers from past management operations and might be characterized by much lower growth rates than would be predicted given their actual size (fig. 3).

To better understand what model component might be responsible for both the observed variance reduction and for underestimation of the higher end of growth range, we re-ran the Monte Carlo analysis on simulated data apportioned into dbh size classes (fig. 4). All classes showed significant differences from their real data counterparts (two-sample t test); while growth was usually overpredicted in medium-sized trees, it was underpredicted in both small and large trees, with the bias in the first category being the most severe (table 4).

The calibration and randomization routines embedded in FVS should partially resolve this issue (Dixon 2002; Stage 1973), but they were not applied here. Our main scope was to suggest SA as a means of preliminary model screening, underlining the inaccuracies of the FVS-SN base growth model when applied to a local dataset. Such framework should be applicable to all cases, and not only for those submodels that may benefit from the thorough calibration routines referenced by Dixon (2002). Moreover, FVS developers themselves later acknowledged as “unreasonable to assume that growth responses in locations with substantially different environmental limitations will be the same. It is more likely the shape of the response surface in these locations, relative to the selected set of predictor variables, will be different. When this is the case, the models should be refit” (Dixon 2002).

Underestimation of diameter growth might affect the final simulation result, both at the individual and at the stand level. For example, density-dependent mortality is triggered by a threshold relative density value (DeRose and others, this proceedings), and in turn mortality intensity depends on simulated relative density of the stand. Underestimation of individual dbh and thus quadratic mean diameter of the stand possibly will result in overpredictions of mean size and density combinations and therefore underpredict competition-induced mortality.

Diameter growth underprediction may be driven by a number of factors, including both assuming excessively severe competition, and a disproportionate influence

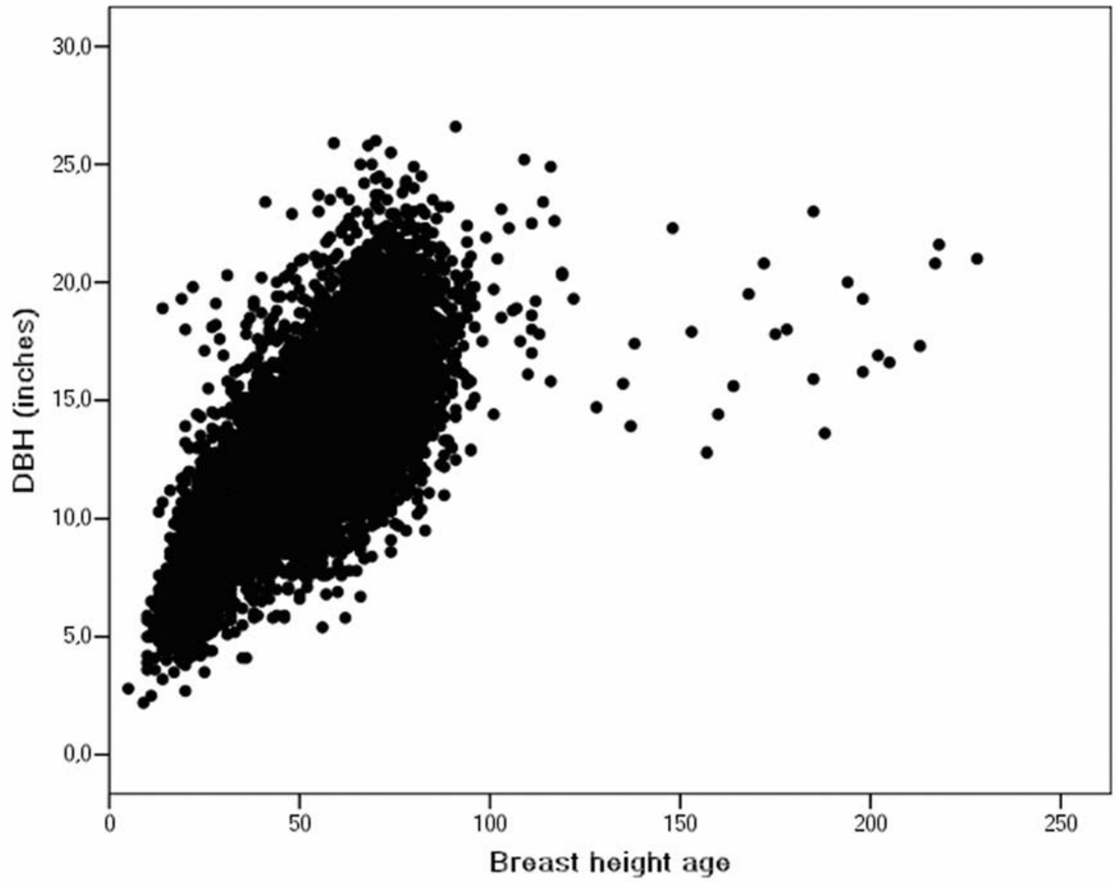

Figure 3—Breast height diameter to breast height age relationship in the sample.

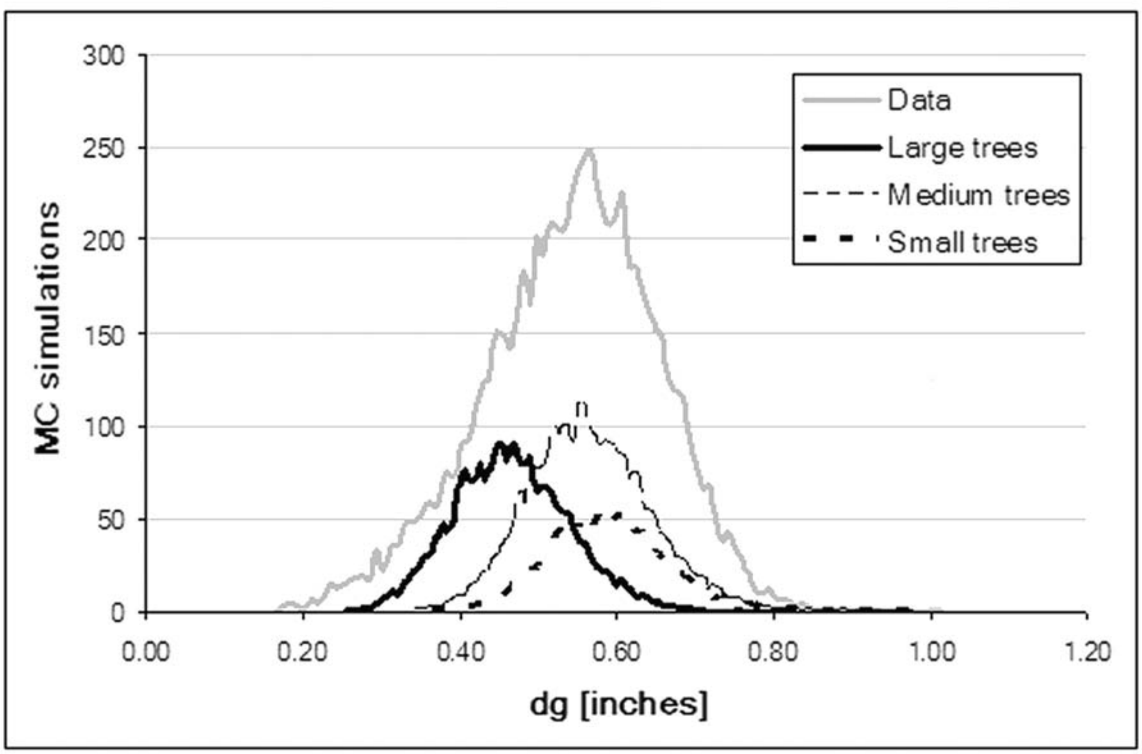

Figure 4—Uncertainty analysis of simulated 5-year diameter increment apportioned into the three dbh size classes (see text for description of size classes), as compared to that measured in the field.

Table 4—Mean and range of 5-year diameter growth (inches) for sample-based simulations (10,000 Monte Carlo runs per size class) as compared to field data. Very small trees: dbh 3 to 5 inches; small: 5 to 10 inches; medium: 10 to 15 inches; large: higher than 15 inches.

Size classes	Simulated data			Fort Bragg inventory	
	Mean	Range	R^2	Mean	Range
	inches	*inches*		*inches*	*inches*
Very small	0.82	0.39–2.58	0.85	0.66	0.16–1.89
Small	0.59	0.36–0.99	0.95	0.75	0.08–2.28
Medium	0.57	0.34–0.98	0.96	0.55	0.08–2.36
Large	0.47	0.25–0.82	0.96	0.50	0.08–1.57

of age-related decline as expressed by the dbh-squared factor. Since the most severe bias affects high increment values of small and medium trees, we hypothesize that the cumulate effect of many competition-related variables in the model could excessively hamper modeled growth.

Sensitivity indices ranking the importance and effect of each input factor are shown by standardized rank regression coefficients (SRRCs; fig. 5) and the Smirnov test index (fig. 6). The signs of all SRRCs (fig. 5) were consistent with expectations for growth behavior. If we exclude the role of forest type coding, which is capable of a large influence on growth prediction in a limited number of cases (when different from longleaf pine type; fig. 6), the most important variable is tree diameter. This is consistent with evidence from the growth modeling literature (see for example Trasobares and Pukkala 2004. Similarly, the FVS-SN variant manual states: "DBH at the beginning of each projection cycle is usually the strongest single statistical determinant of diameter growth during the cycle" (Donnelly and others 2001). However, the role of starting dbh, always preeminent in predicting basal area increment (data not shown), is differentiated when growth output is back-transformed to inside-bark inches of increment.

Large trees showed a very strong negative influence of dbh on increment prediction, an apparent result of the senescence-related dbh-squared term (fig. 5). This is not unexpected, since large trees would mostly be unaffected by competition from neighbors,

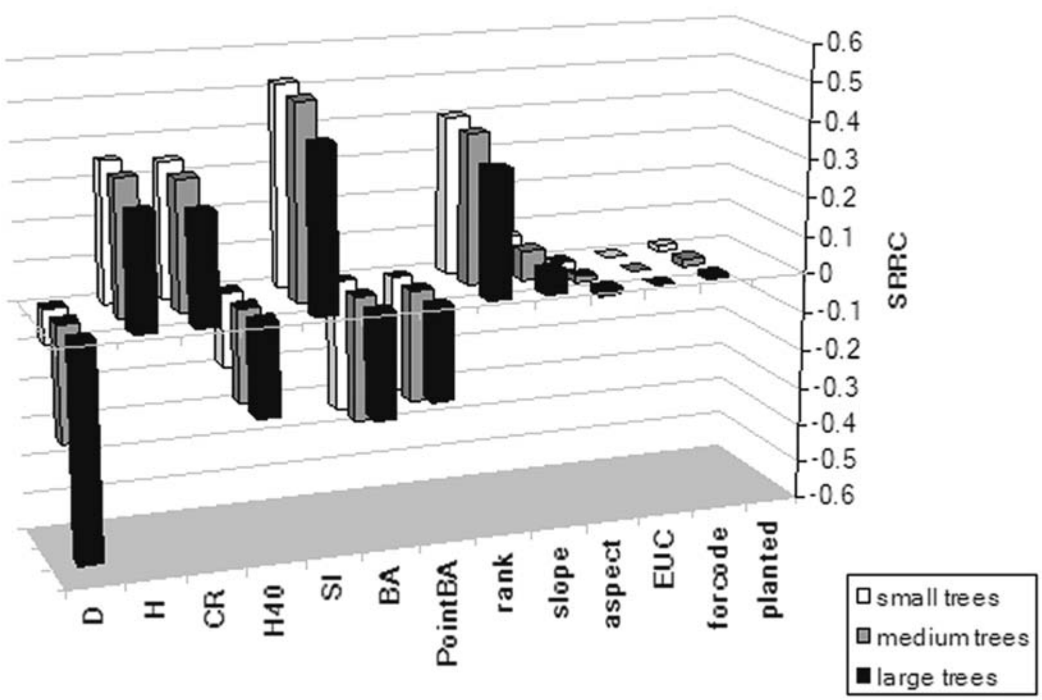

Figure 5—Sensitivity analysis. Standardized rank regression coefficient (SRRC) for input factors of the FVS-SN large tree dbh increment submodel, computed for each dbh size class.

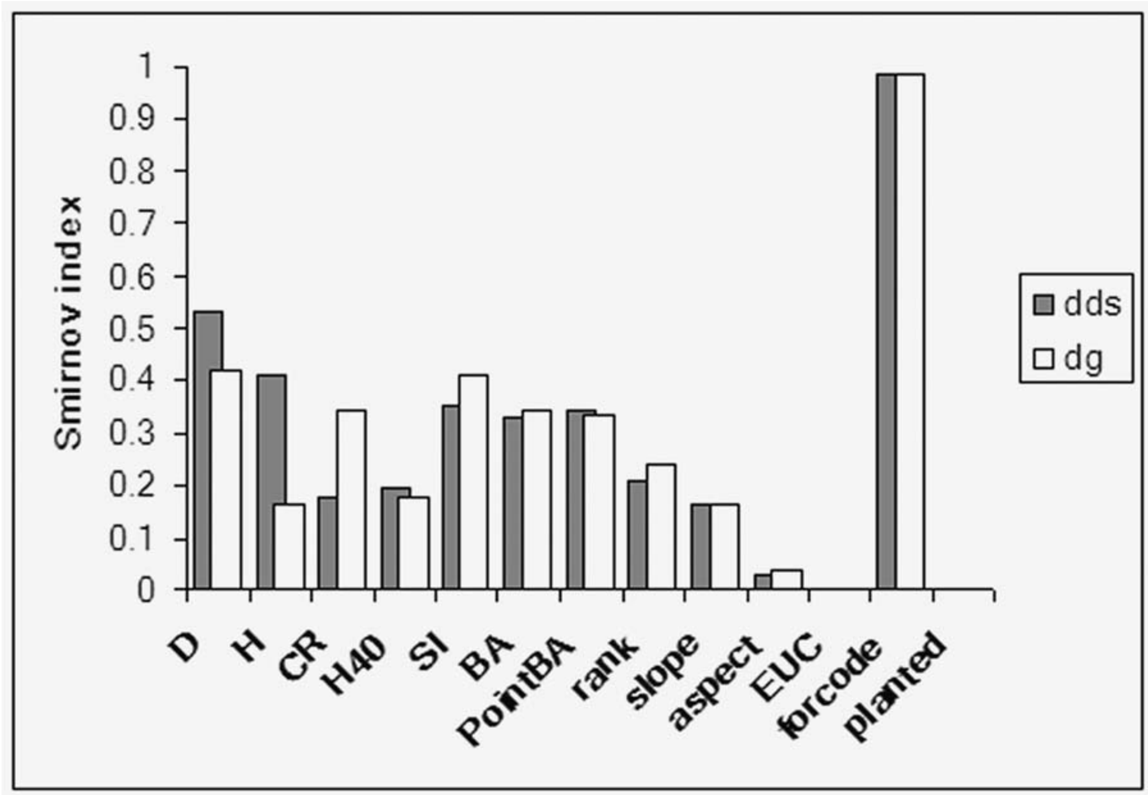

Figure 6—Sensitivity analysis. Smirnov test index for input factors of the FVS-SN large tree dbh increment submodel

and even a more fertile site could not adequately compensate growth decline caused by senescence. Growth of medium and small trees is driven to a greater extent by factors expressing tree and site potential and by competition-related variables. Among factors related to growth potential, site index always took the leading role, with tree height and live crown ratio somewhat less influential (and inherently correlated to tree diameter). If we assumed that the simultaneous action of several competition-related factors in the model is the main reason for growth underpredictions, the ranking operated by SA might be useful to leave out the least important drivers. For example, if just one individual and one stand-scale variable were to be retained, the choice would respectively fall upon individual dbh ranking and stand basal area, which are capable of determining the largest influence on model output among the competitive-related group of predictors.

Topographically related predictor variables such as slope unexpectedly showed a small but significant proportionality to growth, an effect that may be related to site morphology and inherent characteristics of longleaf pine sites. Fort Bragg has rolling terrain and the effects of slope and aspect on forest growth are not readily apparent. Slope position—for example, moist bottomlands vs. dry ridges—is far more likely to influence stand growth than steepness or aspect. Because both high and low moisture extremes are found on sites with relatively low slope values, any effect of slope on growth is likely to be confounded during equation fitting and evaluation.

Conclusions

We propose sensitivity analysis as a preliminary tool to model calibration, and suggest the use of sample-based global sensitivity analysis as a means of ranking the importance of input factors in determining the magnitude of modeled tree growth. Sensitivity analysis can be used to explore model behavior in specific portions of the input space to evaluate biologically sound growth dynamics of different stand components (e.g., partitioning data into size or density classes), and to compare the behavior of alternate model formulations. The analysis could have been done with any submodel of any variant; the flexibility of

SIMLab software represents a strong support to sensitivity analysis of individual FVS submodels and potentially the entire simulation chain.

Once the factors have been ranked in order of importance and the prediction biases have been detected, model developers may simplify model forms in the interest of parsimony or formulate sampling recommendations in order to focus measurement efforts on the most crucial variables. An importance-based ranking of input variables may prove useful in designing complex equations, such as in stepwise approaches to model calibration. After setting up calibrated model runs, a similar analysis to that described in this paper would be useful to show how well the calibrated model performs. Should major model validity problems still exist after a comprehensive calibration, local users would need to look into a refit of the model for local conditions.

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